Order in Distributed Systems

Why Order?

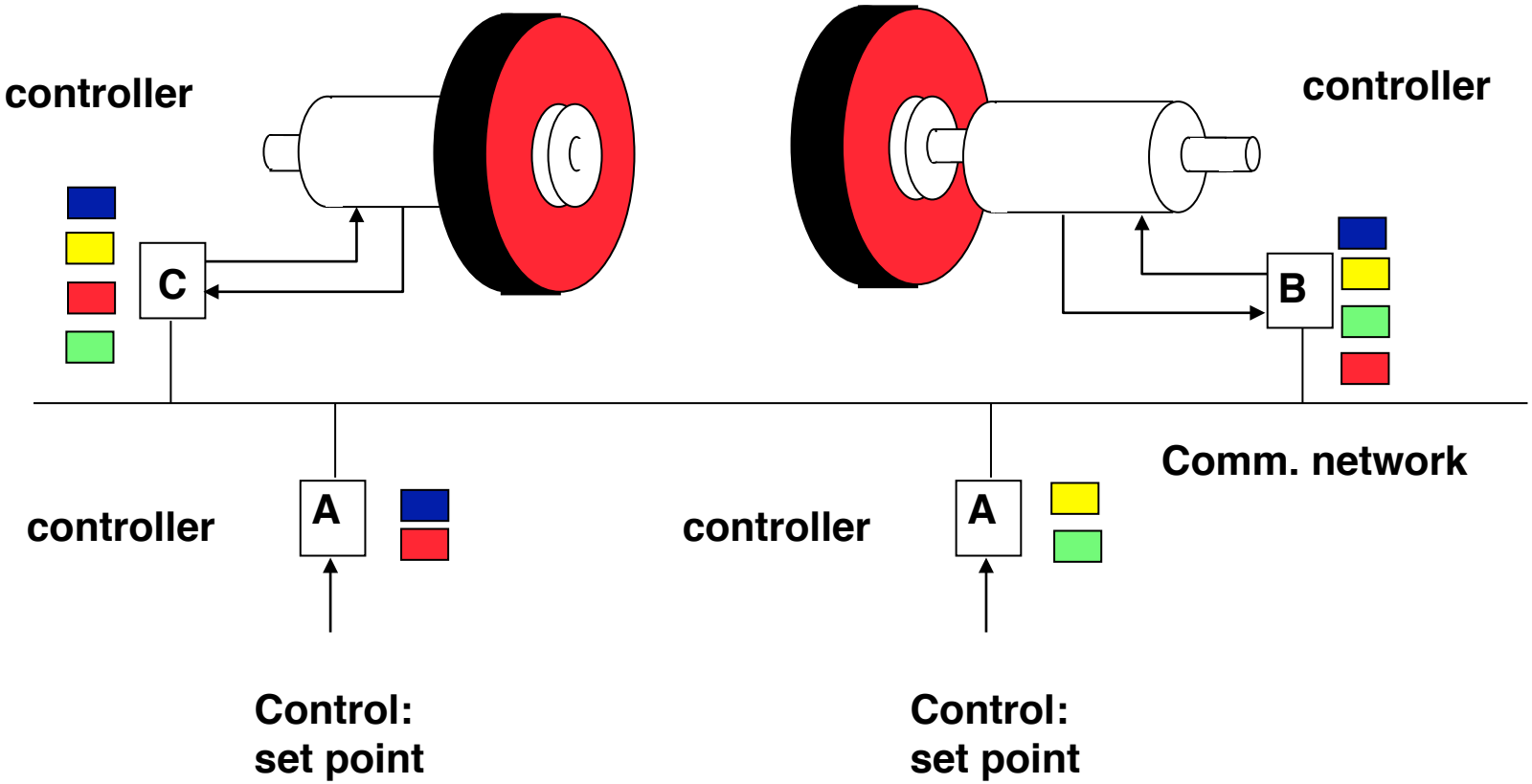
Determine the potential order of events.

- **Determine the cause-effect relationship (causality)in a distributed computation.**

Enforce an ordering policy, i.e. an a priori specified sequence of events.

- **Coordination of joint activities.**

Order is important!



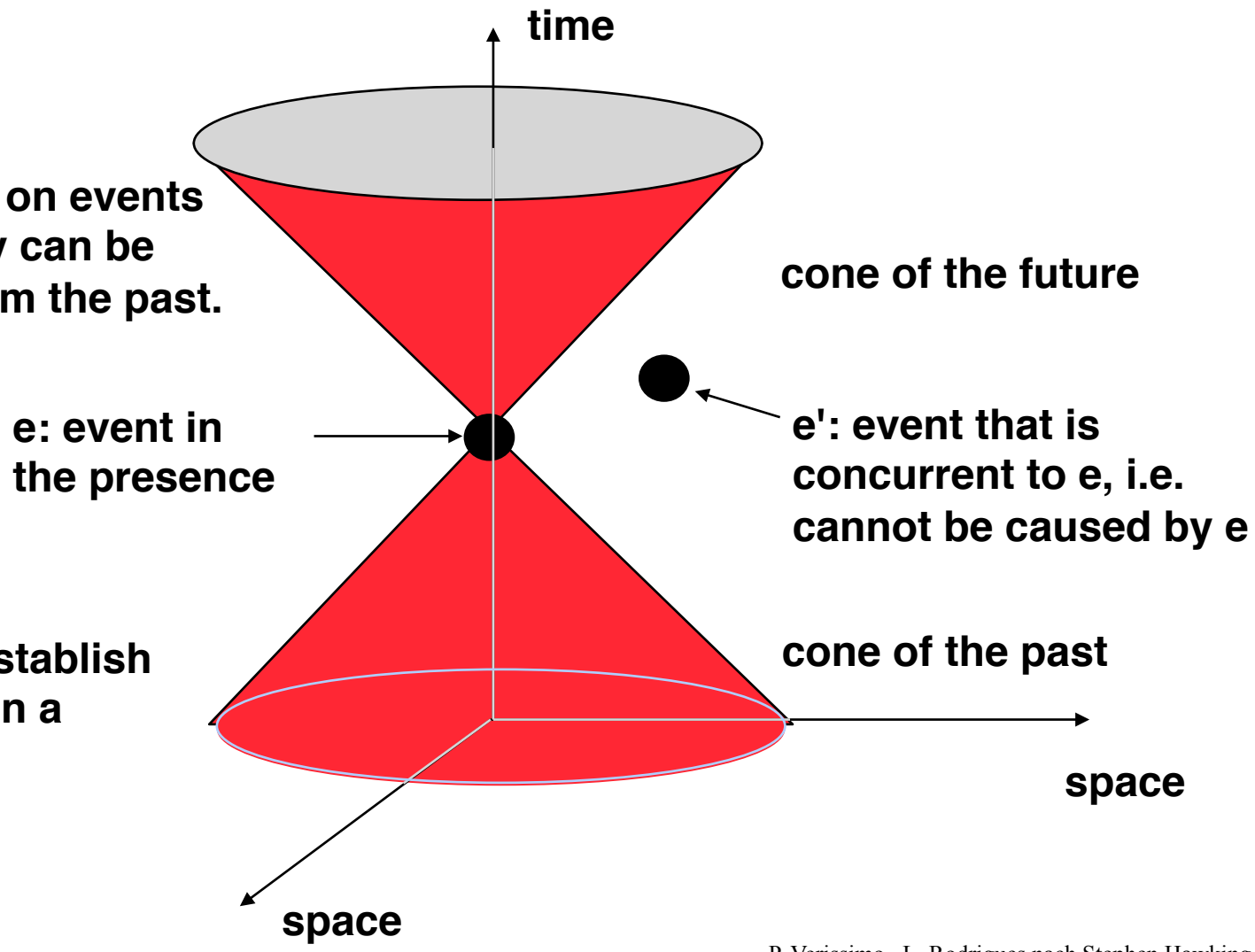
I.

What can be ordered?

In what way order is established in a distributed system?

What can be ordered?

e only has an impact on events in the future and only can be caused by events from the past.

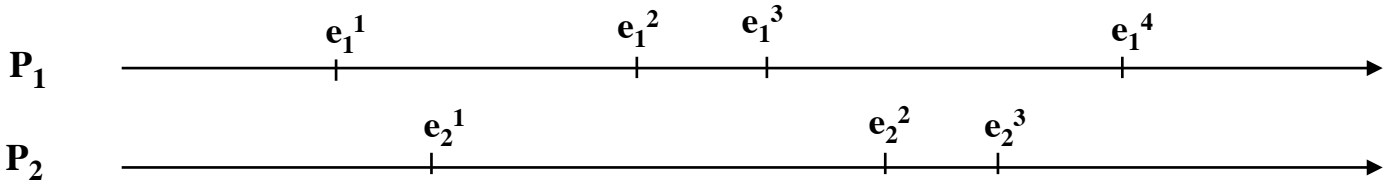


Can we (in principle) establish a total temporal order in a distributed system?

P. Verissimo , L. Rodrigues nach Stephen Hawking

Computational Model

- **A distributed computation is performed as the joint activity of local, sequential processes.**
- **The activity of a local sequential process is modelled as a sequence of events.**



- **An event either is local to a process, i.e. it causes an internal, local state change, or**
- **A computation includes the comminication with another process. This will be modelled by a send and a receive event.**
- **Messages are unambiguous single events, i.e. multiple messages with the same contents sent by the same process will be modelled as multiple individual events.**
- **All models of Data Sharing are abstracted as communication.**

Computational Model

Def.:
The local history of proces p_i is a (possibly infinite) sequence of events $h_i = e_i^1\ e_i^2\ e_i^3\ \ldots\ e_i^n\ \ldots$ (canonical enumeration). It defines a total order of local events.

Def.:
The global history is the set $H = h_1 \cup h_2 \cup h_3 \cup \ldots\ldots \cup h_n$.

Note: The global history does not specify any relative time or order between the elements.

The Precedence Relation

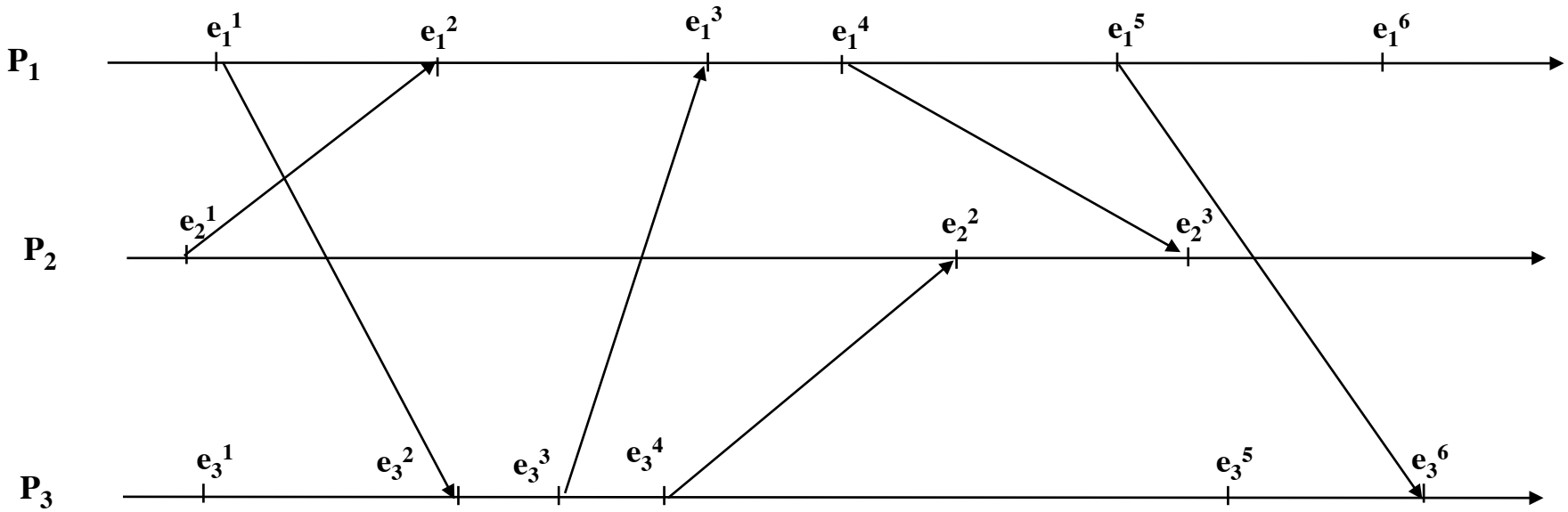
Events in a system can be ordered according to their causal relationship in a cause-effect chain (happens before relation, Lamport 78)).

Def.: Precedence Relation $\rightarrow$
1. for all e_i^k , $e_i^l \in h_i$, $k < l : e_i^k \rightarrow e_i^l$ (h_i is the "history" of process i) (local precedence)
2. If e_i = send (m) and e_j = receive (m) : $e_i \rightarrow e_j$
3. If $e \rightarrow e'$ and $e' \rightarrow e''$: $e \rightarrow e''$ (transitivity)

For concurrent events no causal relationship can be specified, i.e.
neither $e \rightarrow e'$ nor $e' \rightarrow e$ holds. Notation: $e \parallel e'$

A distributed computation can formally be seen as a partially ordered set defined by the tupel (H, $\rightarrow$) where H is the combined History of all processes.

Time-Space Diagram



$e_1^2 \rightarrow e_3^6$

$e_3^1 \parallel e_1^2$

What is the meaning of consistency in a distributed system

?

A system state, that can be established by any possible sequential execution of processes. Causality must be preserved.

Global States and Cuts

Local state:
z_i^k : local state of p_i after the execution of e_i^k

z_i^0 : Initial state of p_i

Global state: $\Sigma = (z_1 , z_2 ,. . . . , z_n)$

Def.: The *Cut C* of a distributed computation is the subset C of the global history H with:

$$C = h_1^{C_1} \cup h_2^{C_2} \cup h_3^{C_3} \cup . . . . . \cup h_n^{C_n} .$$

The tupel of integers $(c_1 , c_2 ,. . . . , c_n)$ represents the resepective index of the last event that has been considered for every process. The set of most recent events $\{ e_1^{C_1}, e_2^{C_2}, e_3^{C_3}. . . . . ., e_n^{C_n} \}$ is called the *front* of the cut.

A global state $(z_1^{C_1}, z_2^{C_2}, z_3^{C_3}. . . . . ., z_n^{C_n})$ is associated with every cut $(c_1 , c_2 ,. . . . , c_n)$.

Runs

Def.: *Run* of a distributed computation:

A *run* of a distributed computation is a totally ordered sequence R, that comprises all events of the global history and is compliant to every local history.

Consistency

1.) A *run* is consistent, if:

for all e, e‘ : (e ∈ C) and (e‘ → e) ⇒ e‘ ∈ C

2.) A consistent (distributed) state is represented by a consistent cut.

Analogies between sequential and distributed computations:

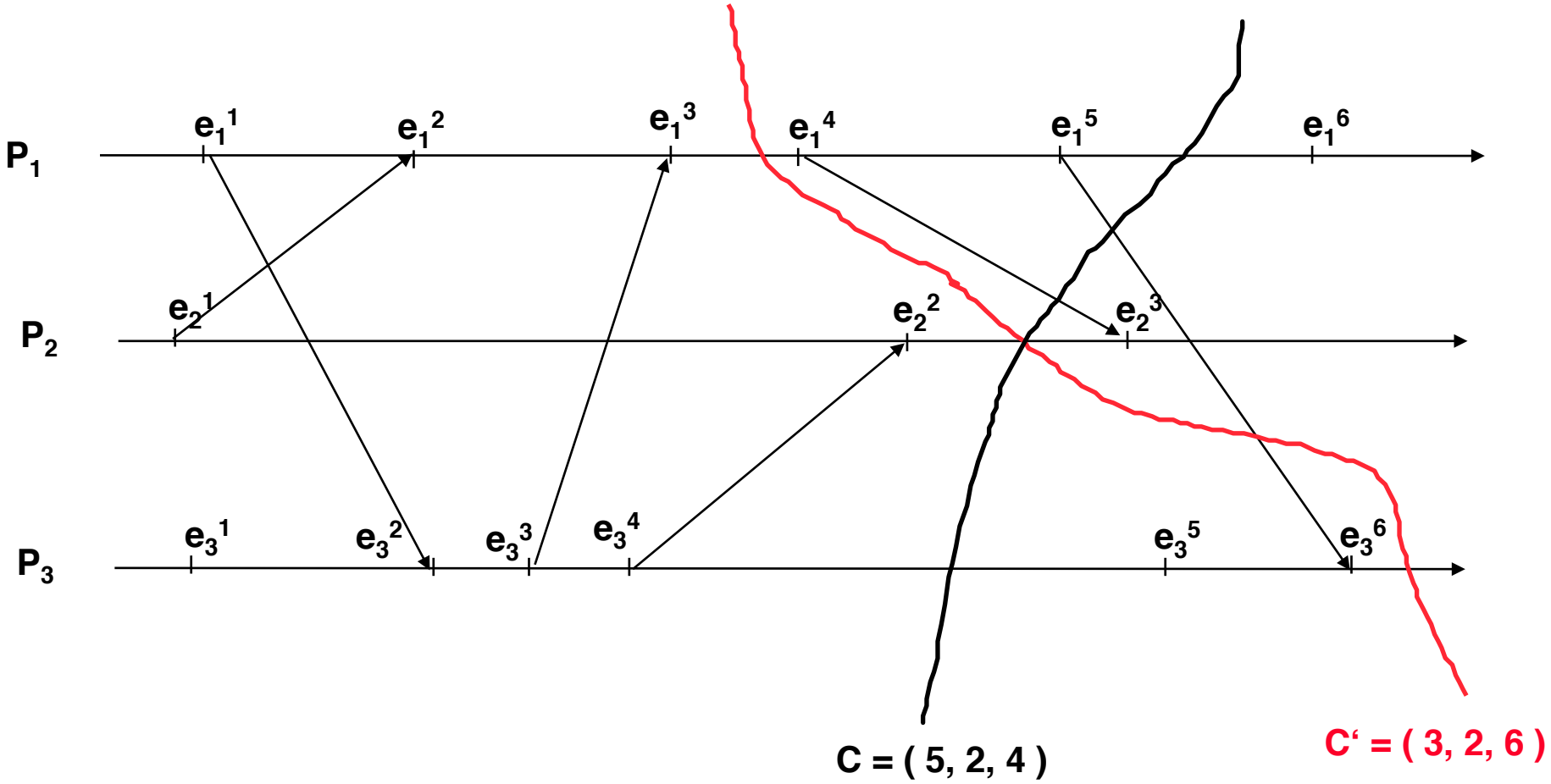
I. The point in time of an event in a sequential computation is equivalent to: The *front* of a consistent cut in a distributed computation.

II. e appears *before* or *after* a certain point in time of a sequential computation is equivalent to: e appears *before* or *after* a cut C, if the event is *left* or *right* of the *front* of C in a distributed computation.

The *run* of a distributed computation is consistent if for all events the following condition holds:

e → e‘ implies that e appears before e‘. (*R* defines a total order)

Runs, global states and cuts



Example: $R = (e_2^1, e_1^1, e_1^2, e_3^1, e_3^2, e_3^3, e_3^4, e_2^2, e_3^5, e_1^3, e_1^4, e_1^5, e_1^6, e_3^5, e_2^2, e_2^3, e_3^6)$

Ordering messages in Distributed Systems

How to order messages ?

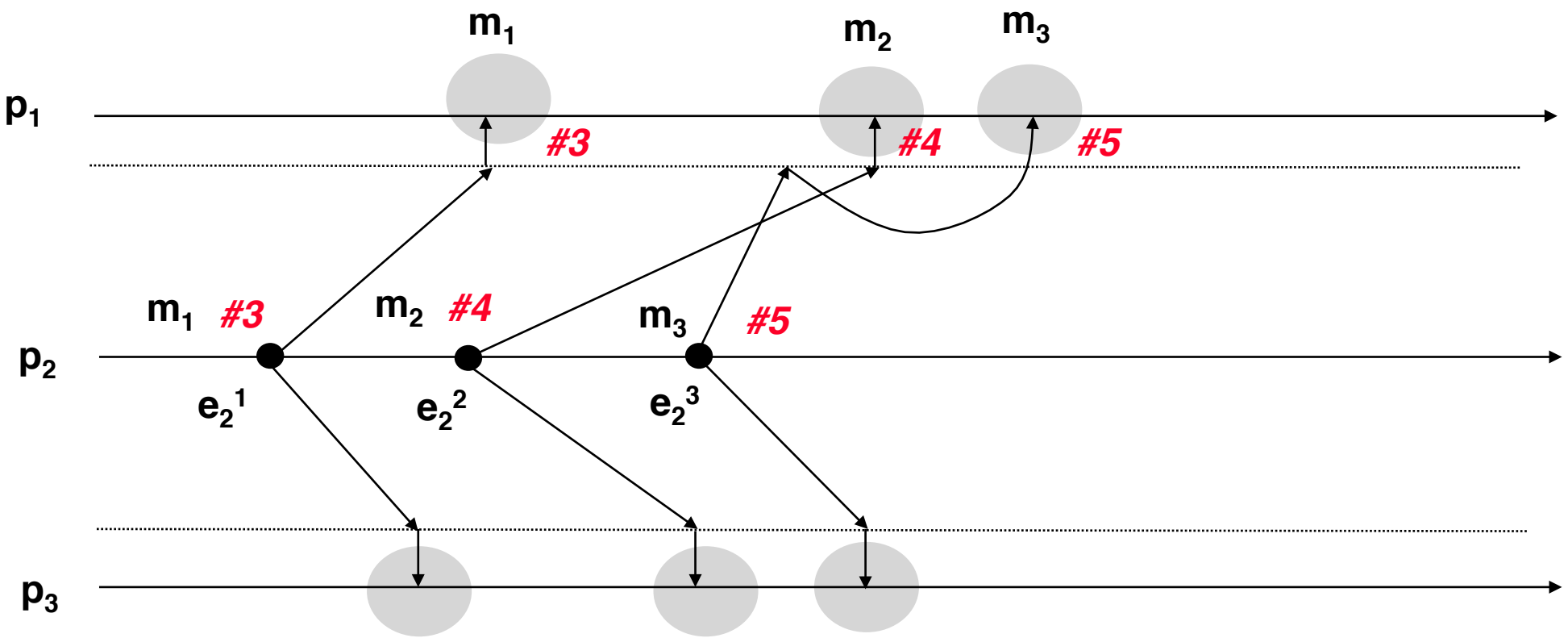
Temporal order: **messages are ordered in a way that the message m_1 sent before message m_2 also will arrive before m_2.**

FIFO .....

CAUSAL .....

TOTAL .....

FIFO-Receive order for pairs of processes



FIFO-order for pairs of processes

- **Idea:** **Receive process reorders the messages.**
- **Approach:** **Distinguish the *receiption* of the message at the node from the *delivery* to an application process**

FIFO-delivery **:** $\text{send}_i(m) \rightarrow \text{send}_i(m') \Rightarrow \text{deliver}_j(m) \rightarrow \text{deliver}_j(m')$

FIFO-D prevents a message from overtaking a message sent later between two processes.

FIFO-order for pairs of processes

Properties:

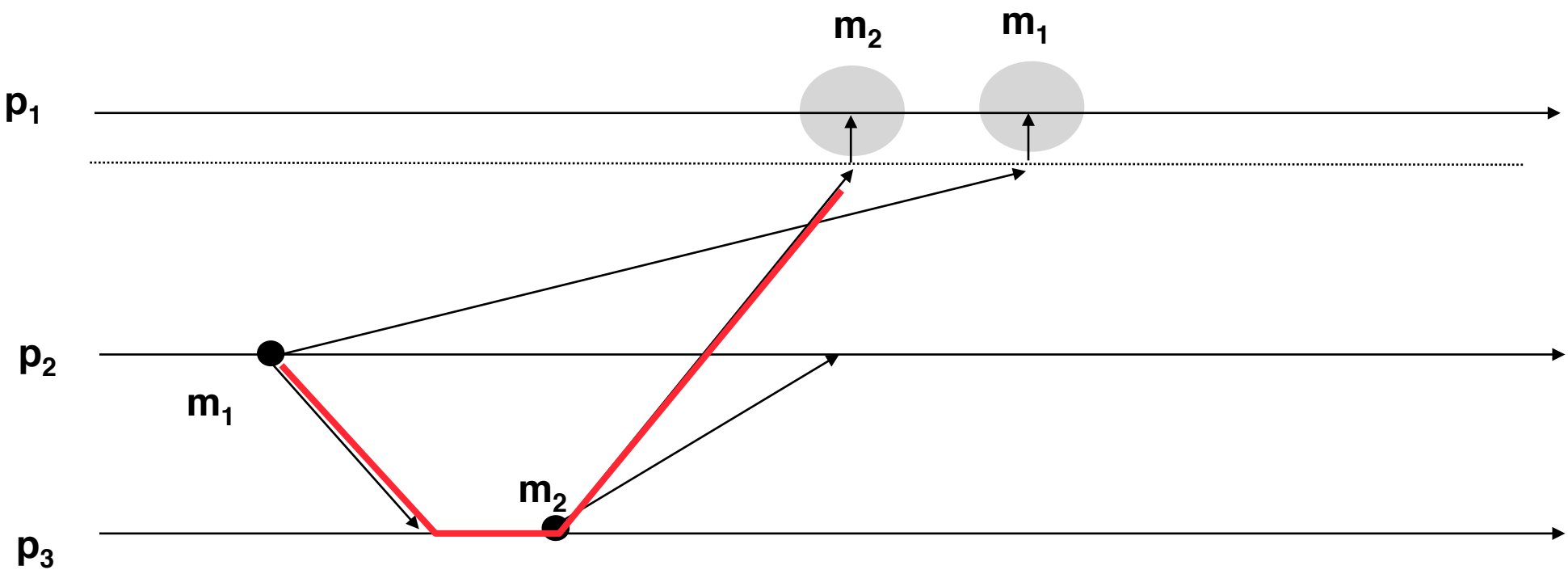
Overhead: Process needs to add a sequence number

FIFO-D is sufficient to guarantee that an observation complies to some run because FIFO-D maintains the order of local events.

BUT:

Because FIFO-D is defined between pairs of processes only it is not sufficient to guarantee that the observation corresponds to a consistent run !

FIFO-D is insufficient



The order of events which p_1 constructs based on the sequence of messages is inconsistent.

FIFO-D doesn't reflect causality for messages sent by different processes!

Causal Delivery

Causal Delivery:
For all messages m, m' and all processes p_i , p_j (send-prozesses) and p_k (receive-prozess) holds:

Causal-D **(CD): $send_i(m) \rightarrow send_j(m') \Rightarrow deliver_k(m) \rightarrow deliver_k(m')$**

CD maintains the global causal order of all messages in the system.

Causal Delivery

Events e und e' may be causally dependent.

To realize causal delivery, we must be able to decide

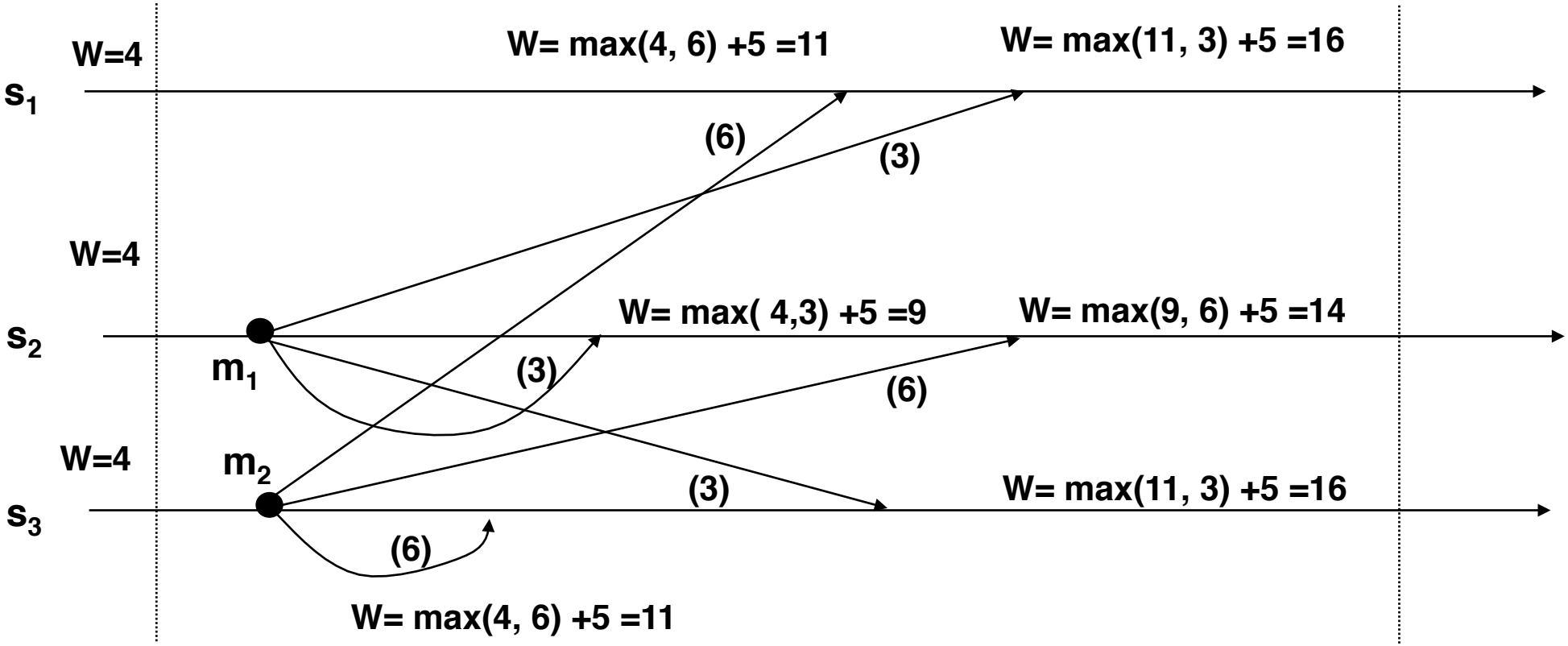
Is there any event e" with the property:

$$e \rightarrow e'' \rightarrow e'$$

?

It is necessary to order the events along causal dependencies.
The temporal sequence of events only defines a potential causal relationship.
Note: Temporal order does not violate causal order.

Is causal order sufficient ?



Every sensor process s_i maintains a variable W that represents a global state e.g. the state of the environment. A new value is calculated from the old value and the messages from the other sensors W_t = max (W_{t-1} , sensor message) + 5

Requirement:

1. **All nodes have the same order of messages**
2. **The order should reflect the causal relationships correctly.**
3. **Concurrent messages have an arbitrary but consistent order.**

How to realize?

Total order

Goal: Observer, which orders all local events in a consisten global stream of events
⇒ produce a *totally ordered* event stream.

Intuitive solution:
Use global time.

Assumptions:
1. All processes have access to a global clock and can take timestamps from that.
2. Communication latencies can be bounded by *d* .

RC(e) is the value of the global clock when the event e occurs.
RC(e) is added as timestamp TS to the message.

Delivery rule:

DR 1 : At time t deliver all received messages in ascending order of the timestamps TS with TS = t - d.

Why is global consistency ensured by DR 1?

Condition I:
The latency of messages is bound by *d*. Therefore, at time t all messages sent before t-*d* have been received. No message sent earlier than t-d will ever be received after t.

Condition II:
The observation is consistent iff the clock condition : e $\rightarrow$ e' $\Rightarrow$ RC(e) < RC(e') holds.
This condition is ensured by the global time.

Disadvantage: **Availability of global time.**
Question: **Can consistency of ordering be achieved without physical time?**

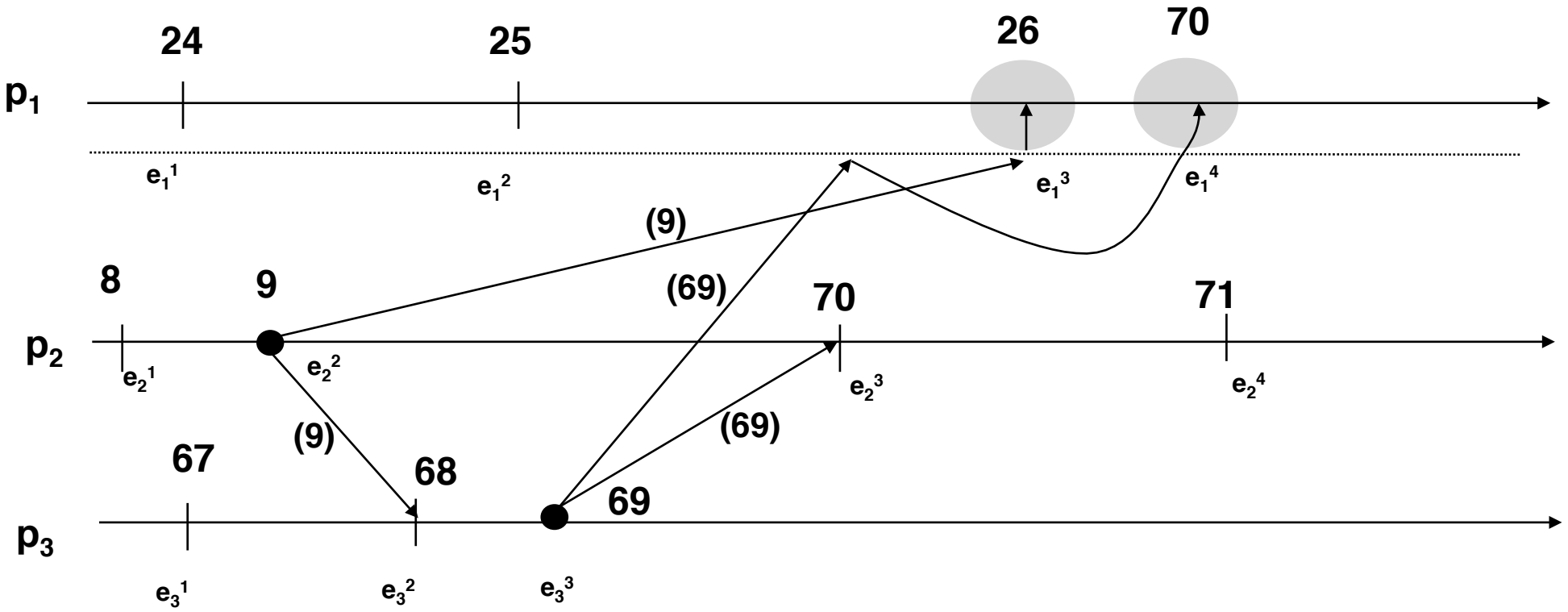
Logic Clocks (Lamport 1978)

Basic Idea: **To achieve a consistent order of messages, we only have to consider the causal relationships. Concurrent messages can be ordered arbitrarily BUT everywhere in the same order.**

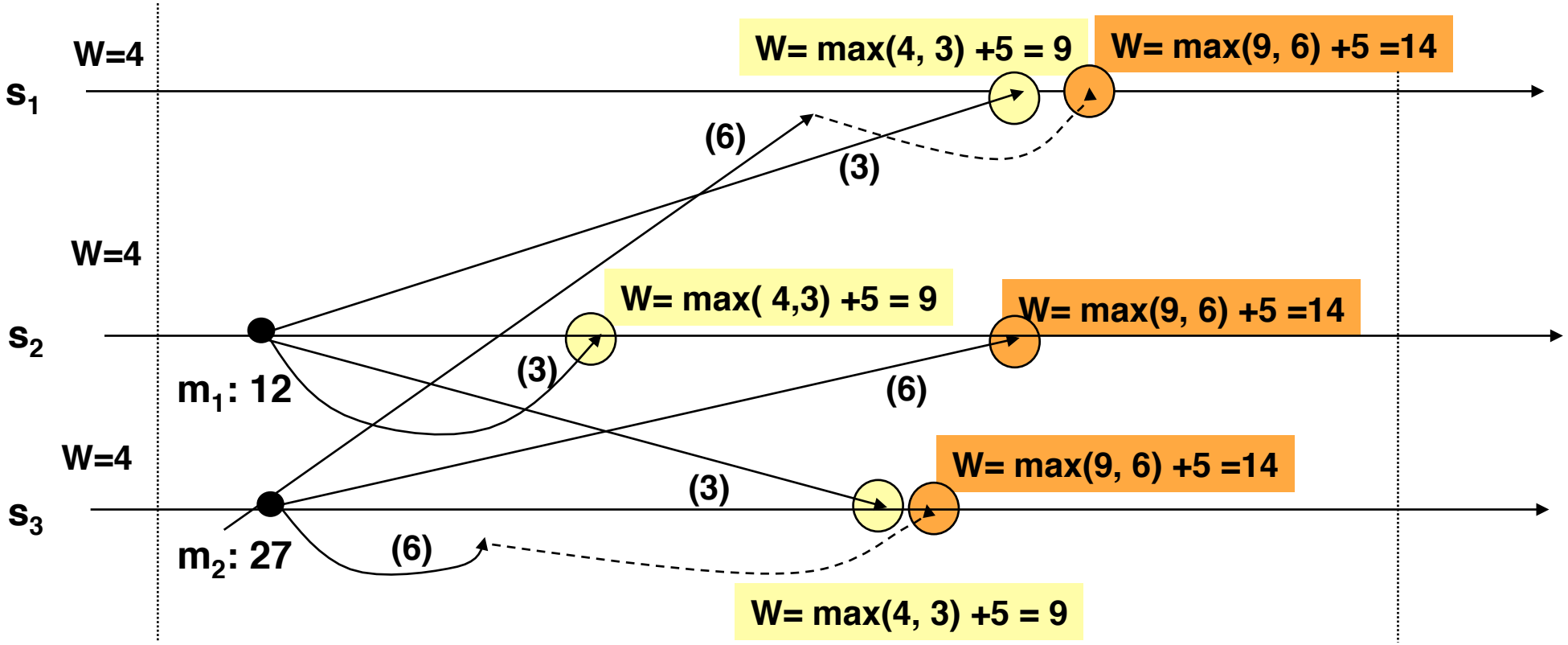
i.e.

The order based on ascending logical time must correspond to the causal order.

Total Order


24
25
26
70
p1
e1 1
e1 2
e1 3
e1 4
(9)
8
9
(69)
70
71
p2
e2 1
e2 2
e2 3
e2 4
(9)
(69)
67
68
69
p3
e3 1
e3 2
e3 3

Total Order


W=4
s_1
W= max(4, 3) +5 = 9
W= max(9, 6) +5 =14
(6)
(3)
W=4
s_2
W= max(4,3) +5 = 9
W= max(9, 6) +5 =14
m_1: 12
(3)
(6)
W=4
s_3
W= max(9, 6) +5 =14
(3)
m_2: 27
(6)
W= max(4, 3) +5 = 9

Logic Clocks

- Every process maintains a variable LC that represents the individual logical clock. LC maps local events on positive intergers.
- $LC(e_i)$: logical clock value of process p_i, when event e_i is generated.
- Every message m that is sent carries the timestamp TS(m), which represents the logical clock value of the sending process.
- Initialization: Before any event is generated, all logical clocks will be reset to "0".
- The following update rule defines the logical clock modification of process p_i when event e_i occurs:

$$LC(e_i) := \begin{cases} LC + 1 & \text{if } e_i \text{ is a local event or a send event} \\ \max\{LC, TS(m)\} + 1 & \text{if } e_i \text{ is a receive event} \end{cases}$$

Properties of Logic Clocks

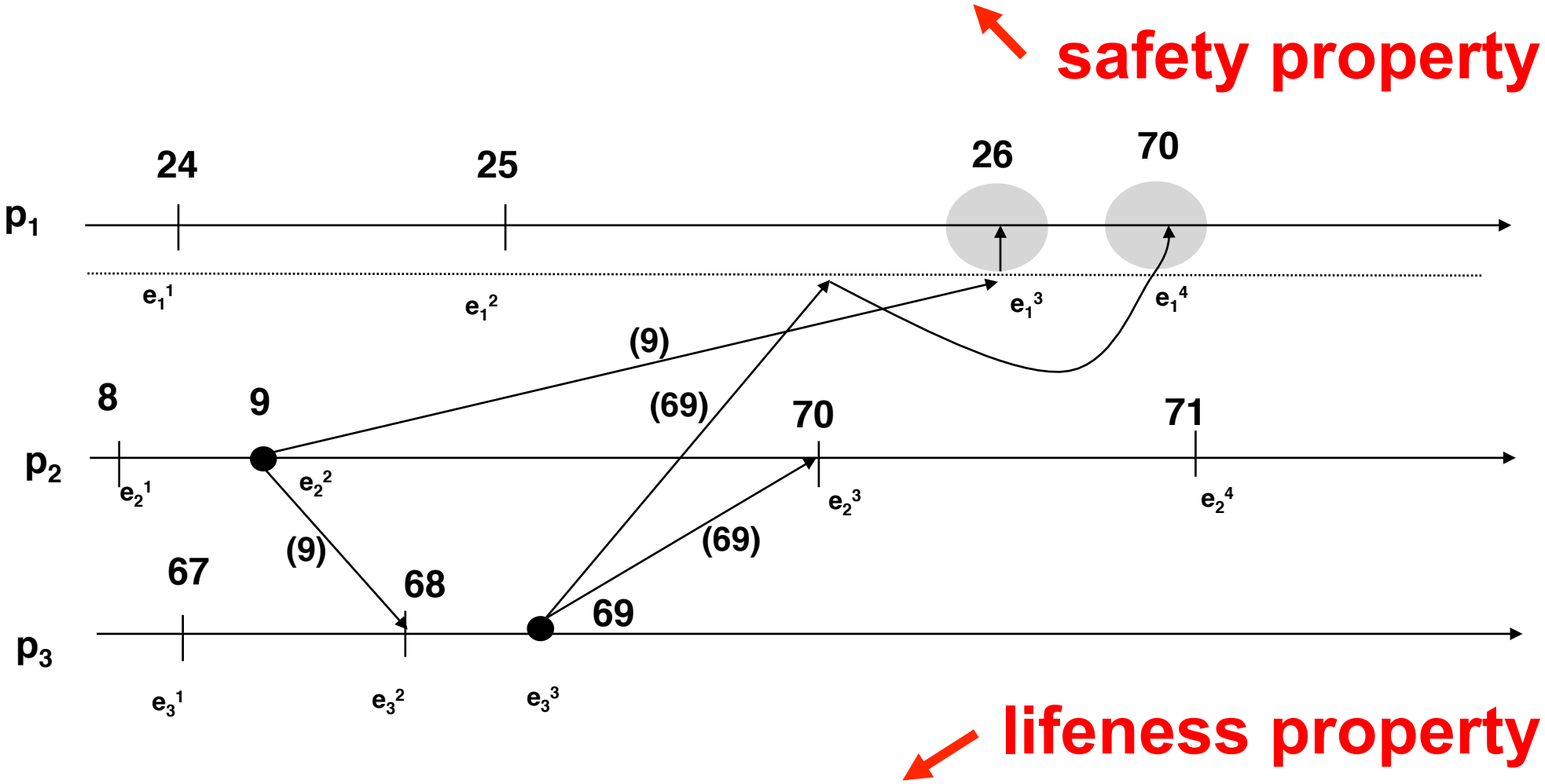
- Local clocks always produce increasing values
- Logic clock values are increasing with respect to causal order
- Logic clocks satisfy the condition : $e \rightarrow e' \Rightarrow LC(e) < LC(e')$.

  This is called the weak Clock Condition because: $LC(e) < LC(e') \not\Rightarrow e \rightarrow e'$

Question:

Are logic clocks sufficient to guarantee consistent observations?

Total order is correctly established by logic clocks



BUT: in an asynchronous system it is impossible to determine when a message may be delivered.

Gap-Detection Property (GDP)

Given the events e und e' with clock values LC(e) and LC(e').
The condition LC(e) < LC(e') holds.

GDP denotes the ability to decide whether there exist an event e'' which satisfies LC(e) < LC(e'') < LC(e')

GDP is needed to guarantee lifeness.

Problem: Find an algorithm with the following properties:

1. All events are totally ordered
2. On the basis of receive events it can be decided when a message can be delivered

Note: Real-time clocks don't solve the problem!

Gap-Detection Property (GDP)

Matrix Clocks
Vector Clocks } **have the GAP detection property**

Synchronous protocols solve the GAP detection problem.

Synchronous Systems

The communication system has a known and bounded maximal message delay *d*.

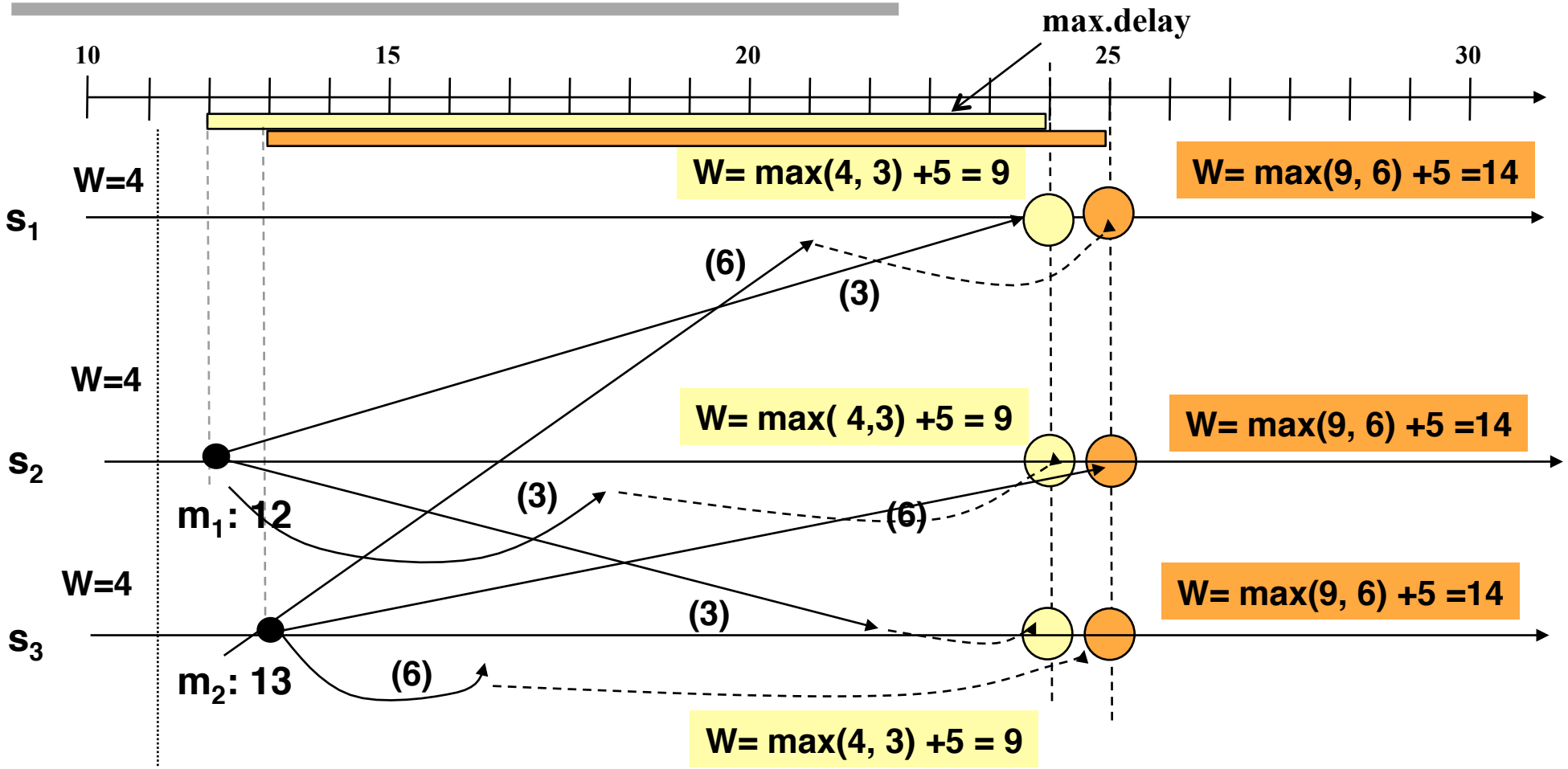
All processes have access to a global real-time clock (RC).

RC(e) is the value of the global clock when event “e” occurs.
RC(e) is added as timestamp TS to the message

Delivery rule:

At time t deliver all messages in ascending order with TS = t - d.

Total Order in a Synchronous System



Temporal order

A message m_1 is temporally preceding a message m_2 if m_1 is sent at least δ before m_2 , i.e. :

$$t(send(m_1)) - t(send(m_2)) > \delta$$

According to this definition, a protocol that delivers messages in temporal order also guarantees causal order.

Synchrony Metrics

Problem # 1:
How big is the max. difference of message propagation of ONE message?

→ Tightness

Problem #2:
How big is the max. difference of message propagation of DIFFERENT messages?

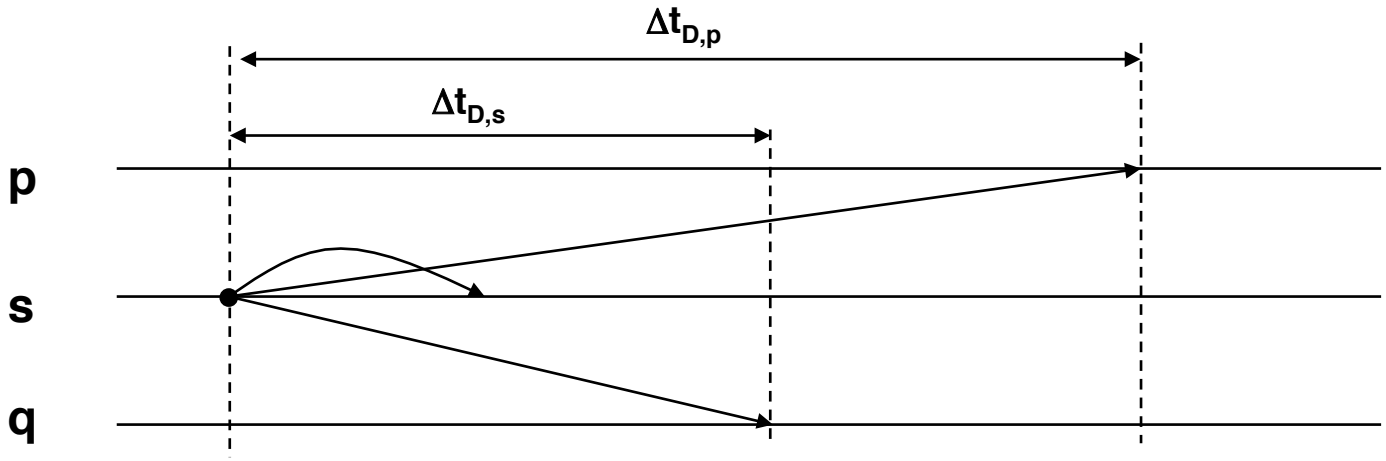
→ Steadiness

Synchrony Metrics

Definition: Delivery time of a message

$\Delta t_{D,p} = t(\text{deliver}_p(m)) - t(\text{send}(m))$

$\Delta t_{D,p}$ **: Interval between the send event of message m its delivery at prozess p**



Synchrony Metrics

Tightness τ

Definition: Tightness

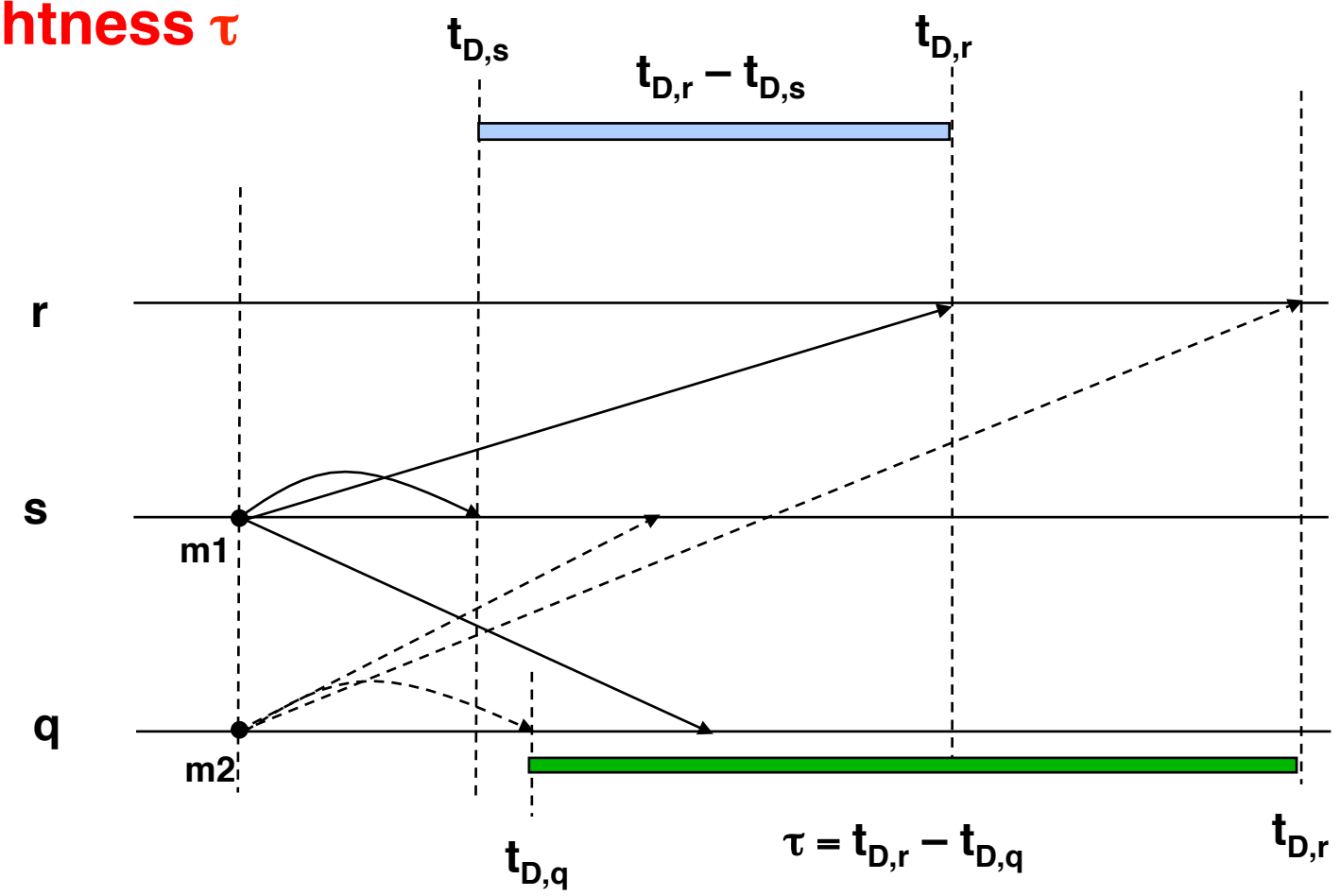
$\tau = \max_{m,p,q} (t_{D,p} - t_{D,q})$

for every message m, τ is the maximal difference of transmission times that occurs for arbitrary receivers p and q.

Tightness is a measure for the difference of transmission times of ONE message to DIFFERENT nodes.

Synchrony Metrics

Tightness τ



Synchrony Metrics

Steadiness σ

Definition: Steadyness

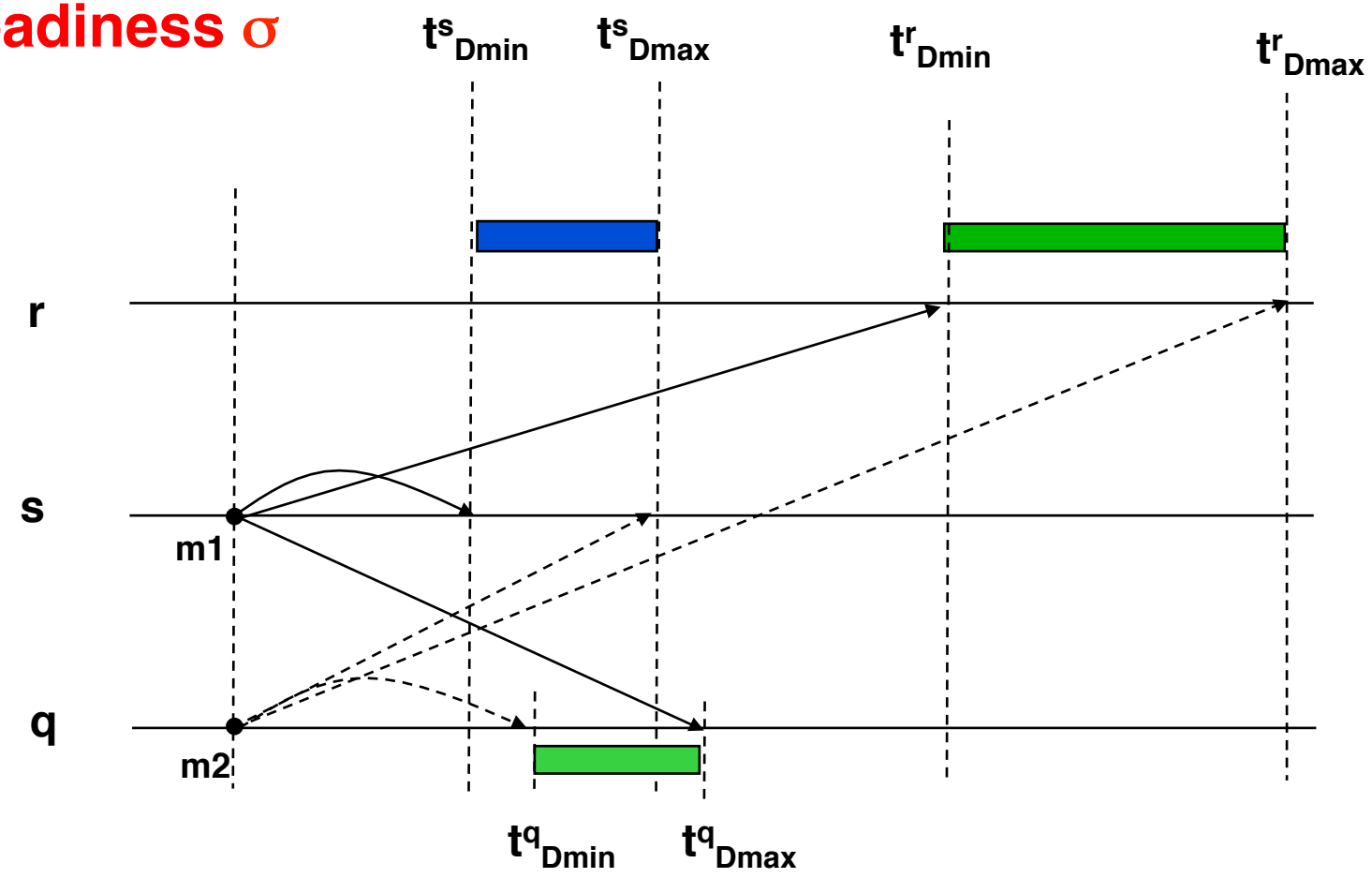
$\sigma = \max_p (t^p_{Dmax} - t^p_{Dmin})$

σ is the maximal difference that can be observed between the maximum t^p_{Dmax} and the minimum t^p_{Dmin} delivery times of different message (to some process p).

Steadiness measures the maximal difference of delivery times of DIFFERENT mesages to the same process.

Synchrony Metrics

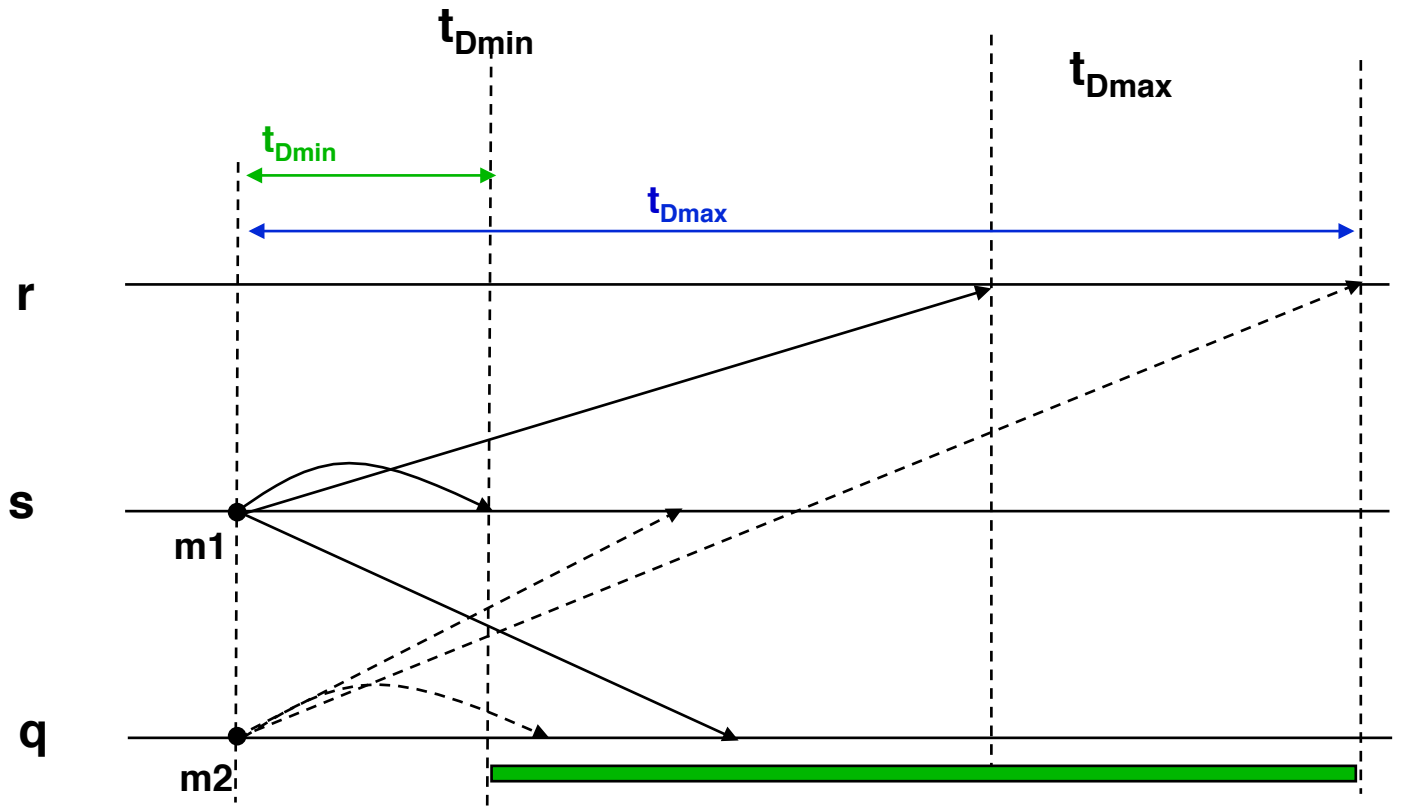
Steadiness σ



$$\sigma = \max_p (t^p_{Dmax} - t^p_{Dmin})$$

Synchrony Metrics

Temporal uncertainty ε



$$\varepsilon = \max_{p,q} (t^p_{Dmax} - t^q_{Dmin})$$

Temporal uncertainty describes the absolute max difference between the minimal delivery time and the maximal delivery time of all messages.